The Lepton Sector of a Fourth Generation

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Abstract

In extensions of the standard model with a heavy fourth generation one important question is what makes the fourth-generation lepton sector, particularly the neutrinos, so different from the lighter three generations. We study this question in the context of models of electroweak symmetry breaking in warped extra dimensions, where the flavor hierarchy is generated by the localization of the zero-mode fermions in the extra dimension. In this setup the Higgs sector is localized near the infrared brane, whereas the Majorana mass term is localized at the ultraviolet brane. As a result, light neutrinos are almost entirely Majorana particles, whereas the fourth generation neutrino is mostly a Dirac fermion. We show that it is possible to obtain heavy fourth-generation leptons in regions of parameter space where the light neutrino masses and mixings are compatible with observation. We study the impact of these bounds, as well as the ones from lepton flavor violation, on the phenomenology of these models.

1 Introduction

Perhaps the simplest extension of the standard model is to allow for a chiral fourth generation. An obvious objection to this addition is the fact that the number of light neutrinos is 3, as accurately measured for instance in e^+e^- collisions [1]. Thus, the fourth-generation neutrino should be at least heavier than $M_Z/2$, which appears unnatural. Similarly, the fourth-generation charged lepton should have masses of the $O(100)$ GeV. It is possible to realize this situation in theories with one compact extra dimension with an AdS metric [2]. In theories of electroweak symmetry breaking in these backgrounds the Higgs field must be localized close to the so-called infra-red (IR) brane in order to address the hierarchy problem. The fermion mass hierarchy, can then be naturally realized by localizing the zero-mode fermions close to or away from the IR brane. Then, if a fourth generation is added in this scenario, it suffices to localize its zero modes close to the IR brane in order for them to have large enough masses. This was the construction used in Ref. [3] where, in addition, the resulting strong couplings of fourth-generation quarks to KK gluons were used to trigger electroweak symmetry breaking [4]. Since a heavy fourth generation must have large couplings to the dynamics responsible for electroweak symmetry breaking, it is natural to consider it in association with strongly coupled TeV-scale physics. The phenomenology of the strongly coupled quark sector of a fourth generation was studied in this context in Ref. [5]. However, it is interesting to study the lepton sector of a fourth generation in warped extra dimensions independently of the origin of the Higgs sector. Recently, the constraints on a standard model fourth generation have been re-examined [6], leading to the realization that a chiral fourth generation is allowed by electroweak precision constraints as long as the Higgs is heavier than in the standard model fits, and the mass differences in the isospin doublets are kept below M_W. Constraints might be even weaker in extensions of the standard model with a chiral fourth generation.

In this paper we consider theories with one compact extra dimension with AdS metric (AdS$_5$) assuming the presence of four chiral generations in the bulk. For the most part, we will not need to assume a specific form of the Higgs sector, as long as the physical Higgs is localized close to the IR. Thus, most of our discussion applies to the generic model with an IR-localized Higgs, as well as to composite Higgs models [7] and the fourth-generation condensation scenario [3]. Our goal is to show that the parameter space in the lepton sector of these models with a heavy chiral fourth generation results in the correct pattern of neutrino masses and mixings, for acceptable values of the fourth-generation lepton masses and their mixing with the lighter generations.

The resulting allowed parameter space in couplings and masses can then be used to study the phenomenology of these fourth generation leptons at colliders, particularly at the Large Hadron Collider (LHC).

In the next section we describe the 5D model of fourth generation leptons. We study the spectrum of the zero-mode leptons, as well as the constraints imposed by light neutrino masses and mixings, and by the mixing of the fourth generation to the lighter leptons. In Section 3 we consider the constraints arising from flavor-changing processes, such as $\mu^- \to e^- e^+ e^-$ and $\mu^- - e^-$ conversion in nuclei. In Section 2.1 we list the possible representations that leptons can take in the bulk gauge theory, which we take to be $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X$ [8]. This will be of use when studying the phenomenology of the lepton sector, which is done in Section 4, where we define the allowed parameter space and show that a typical occurrence in these models is the appearance of light Kaluza-Klein (KK) modes of leptons. Here we also show the couplings of the zero-mode fourth-generation leptons to gauge bosons, which will determine their collider phenomenology. We conclude in Section 5.

2 Fourth Generation Leptons in AdS$_5$

The model we consider here originates from a five-dimensional theory in an AdS background with the metric defined by [2]

$$ds^2 = e^{-2\sigma(y)}\, \eta_{\mu\nu}\, dx^\mu\, dx^\nu - dy^2 \ , \tag{1}$$

where $\sigma(y) = k|y|$ and k is the AdS curvature, y is the coordinate in the extra dimension and it is bound to be in a segment between 0 (ultra-violet or UV) and L (IR). The electroweak gauge group in the bulk is $SU(2)_L \times SU(2)_R \times U(1)_X$, with the hypercharge defined as $Y = T_R^3 + X$, where T_R^3 is the $SU(2)_R$ isospin and X is the charge under $U(1)_X$ [8]. The bulk gauge symmetry is broken down to the SM gauge group by boundary conditions in the UV which lead to $SU(2)_R \times U(1)_X \to U(1)_Y$. However, as we will show below, many properties of the lepton model are independent of the choice of SU(2)$_R$ embedding for the fermions.

We consider four generations of leptons propagating in the 5D bulk:

$$\xi_i^l \ , \quad \xi_i^\nu \ , \quad \xi_i^e \ ; \quad i = 1, \ldots 4. \tag{2}$$

with corresponding bulk 5D masses $c_{l,\nu,e}^i$ in units of the AdS curvature k . The boundary conditions allow us to obtain a spectrum of zero modes that reproduces the SM lepton spectrum,

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plus a SM-like fourth generation with the $SU(2)_L$ doublets arising from ξ^l and singlets from $\xi^{\nu,e}$.

Since the $SU(2)_R \times U(1)_X$ gauge symmetry is broken to $U(1)_Y$ by boundary conditions in the UV, a Majorana mass for bulk fermions transforming under $SU(2)_R$ can only be added on the UV boundary. So we introduce a UV-localized Majorana mass term for the right-handed neutrino of the form

$$\mathcal{S}_0 = \int dx^4 dy \sqrt{g} \ \left(-\frac{1}{2} \bar{\xi}_R^{\nu c} M_{UV} \xi_R^{\nu} + \text{h.c.} \right) \frac{\delta(y)}{\Lambda_{UV}} \ , \tag{3}$$

where M_{UV} is a 4×4 matrix, generation indexes are suppressed and Λ_{UV} is the UV cutoff, typically the Planck mass M_P.

We also consider Dirac masses for the leptons in the 5D bulk. These arise from their 5D Yukawa interactions with the Higgs doublet

$$\mathcal{L}_5 = -\lambda_\nu \bar{\xi}^l H \xi^\nu - \lambda_e \bar{\xi}^l H \xi^e + \text{h.c.} \ , \tag{4}$$

with $\lambda_{\nu,e}$ being 4×4 matrices and the Higgs H localized towards the IR. In the scenario of Ref. [3] the Higgs arises from the condensation of the zero-mode quarks of the fourth generation, but our treatment of the fourth generation lepton sector is general, and will remain independent of the details of the Higgs sector unless specified.

In general we will see that the zero-mode spectrum furnishes a complete representation of the SM gauge symmetry instead of the full 5D gauge symmetry, since in most cases $SU(2)_R$ is only broken in the UV boundary. The only exception is the case where the 5D field ξ^ν is a singlet under all gauge interactions, thus allowing a Majorana term in the IR. We will treat this case separately. but in general our results will be independent of the $SU(2)_R$ embedding unless we state otherwise.

The mixing of the zero-modes with the KK modes is not important for the discussion of the zero-mode spectrum, but its effects will be considered in Section 4. After the Higgs acquires a vacuum expectation value (VEV), integrating over the extra dimension Eqs. (3) and (4) leads to

$$\mathcal{L}_{\text{mass}} \ = \ -\frac{1}{2} \bar{\nu}_R^c \ M_{RR}^\nu \ \nu_R - \bar{\nu}_L M_{LR}^\nu \nu_R - \bar{e}_L M_{LR}^e e_R + \text{h.c.} \ , \tag{5}$$

where generation indexes are understood, and for notational simplicity we have hidden the super-index (0) labeling the zero-modes. The mass matrices M_{RR} and $M_{LR}^{e,\nu}$ depend on the Higgs and fermionic zero-mode wave functions and will be computed in the next section.

In order to obtain the observed spectrum and mixings, we have to select the localization of the zero-modes in the extra dimension. As usual in 5D theories, the localization of the zero-modes is determined by the bulk mass parameters $c^i_{l,\nu,e}$ in such a way that, with $\mathcal{O}(1)$ variations in them, we can change the localization from the UV to the IR boundary [9, 10]. Since left-handed leptons form a $SU(2)_L$ doublet, within each generation e_L and ν_L must have the same localization. In addition, to obtain a pattern of neutrino mixings among the light generations compatible with observations, the overlap between the wave-functions of the left-handed zero-mode leptons and the Higgs has to be of the same order for the first three generations. This can be done, for example, by choosing localizations towards the IR, with $-1/2 < c^i_l < 1/2$. However, since in the present model the fourth generation is also localized towards the IR, this scenario would lead to large mixings of the three light generations with the fourth generation. To avoid this, we will localize the left-handed fermions of the first three generations near the UV boundary, $c^i_l \sim 0.6$. In this case the left-handed leptons have an exponentially suppressed overlap with the Higgs. Thus, to obtain same order Yukawas for the light generations, we impose that all the left-handed bulk leptons leading to the light generations have the same localization. This corresponds to

$$c^i_l = c_l , \qquad i = 1, 2, 3 . \tag{6}$$

The requirements on the right-handed neutrinos of the first three generations are that they have to be somewhat UV-localized but not too close to the UV so that their Majorana mass is not too large ($\sim M_P$) compared to the needed value for a successful see-saw mechanism. We will next discuss the specific values of c^i_ν that satisfy this constraint, but for now we can see that in order to obtain a single effective Majorana mass scale in the effective 4D theory of three generations of light neutrinos, we need to impose $c^i_\nu = c_\nu$ for $i = 1, 2, 3$. These assumptions can be cast as the presence of a global family symmetry in the 5D bulk involving the first three generations [11, 12]. Finally, to obtain a heavy fourth generation of leptons we allow l^4_L and ν^4_R to have different localizations from the ones chosen above for $i = 1, 2, 3$, which as we show below must be toward the IR brane. Throughout we consider anarchic Yukawas $\lambda_{\nu,e}$, meaning that all the entries of these matrices are of the same order: $\lambda^{ij}_{\nu,e} = \mathcal{O}(1)$, for $i, j = 1, 2, 3, 4$. Also for simplicity, we choose the UV-localized Majorana mass matrix as $M_{UV} = M_R \to M_R \mathbf{1}$, with M_R a number of order $\sim M_P$.

2.1   5D Lepton Embeddings

To complete the 5D model, we consider here the possible embedding of the lepton sector in the 5D bulk gauge theory $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X$. As we will see in Section 4, some phenomenological details depend on such embeddings. On the other hand, there are some generic features of the model which are mostly independent of the chosen lepton representation in the 5D bulk. We start by pointing out that the Higgs is a $(\mathbf{2},\mathbf{2})_\mathbf{0}$, with $\langle H \rangle \propto \mathbf{1}_{\mathbf{2}\times\mathbf{2}}$. Therefore, the requirement that the mass term (4) be a singlet, determines the transformation properties of the leptons.

Embedding 1
A possible choice of bulk lepton representation under $SU(2)_L \times SU(2)_R \times U(1)_X$ can be

$$\xi_i^l = (\mathbf{2},\mathbf{1})_{-1/2}; \qquad \xi_i^\nu = (\mathbf{1},\mathbf{2})_{-1/2}; \qquad \xi_i^e = (\mathbf{1},\mathbf{2})_{-1/2}. \tag{7}$$

The boundary conditions can be schematically written as

$$
\begin{aligned}
\xi^l &= \left[\; \xi_L^l(++) \;\; \xi_R^l(--) \; \right], & \xi^\nu &= \left[\; \xi_L^\nu = \left[\begin{array}{c} \nu_L'(--) \\ e_L'(+-) \end{array} \right] \;\; \xi_R^\nu = \left[\begin{array}{c} \nu_R(++) \\ e_R'(-+) \end{array} \right] \right], \\
\xi^e &= \left[\; \xi_L^e = \left[\begin{array}{c} \nu_L''(+-) \\ e_L''(--) \end{array} \right] \;\; \xi_R^e = \left[\begin{array}{c} \nu_R''(-+) \\ e_R(++) \end{array} \right] \right],
\end{aligned}
\tag{8}
$$

where $(++)$ refers to Newman boundary conditions on both the UV and the IR branes, $(--)$ to Dirichlet boundary conditions, etc. If we consider the Higgs localized in the IR, Eq. (4) leads to

$$\mathcal{L}_{\text{mass}} = \bar{\xi}_L^l (m_\nu \xi_R^\nu + m_e \xi_R^e)|_{L_1} + \text{h.c.} = [\bar{\nu}_L(m_\nu \nu_R + m_e \nu_R'') + \bar{e}_L(m_\nu e_R' + m_e e_R)]|_{L_1} + \text{h.c.} , \tag{9}$$

with $\xi_L^{lt} = (\nu_L, e_L)$ and $m_{e,\nu} = \lambda_{e,\nu} v/\sqrt{2}$.

Embedding 2
Another possibility is

$$
\xi^l = (\mathbf{2},\mathbf{2})_{-1} = \left[\begin{array}{cc} L_L(++) & L_R(--) \\ L_L'(-+) & L_R'(+-) \end{array} \right]; \tag{10}
$$

$$
\xi^\nu = (\mathbf{1},\mathbf{3})_{-1} = \left[\; \xi_L^\nu = \left[\begin{array}{c} \nu_L''(--) \\ e_L'(+-) \\ \chi_L(+-) \end{array} \right] \;\; \xi_R^\nu = \left[\begin{array}{c} \nu_R(++) \\ e_R'(-+) \\ \chi_R(-+) \end{array} \right] \right]; \tag{11}
$$

$$
\xi^e = (\mathbf{1},\mathbf{1})_{-1} = \left[\; e_L''(--) \;\; e_R(++) \; \right], \tag{12}
$$

where L and L' are the $SU(2)_L$ doublets with $T^{3R} = \pm 1/2$ respectively, contained in ξ^l. The boundary conditions for L account for the zero-mode SM doublet, whereas L' does not have zero-modes and gives rise to exotic KK leptons with electric charge $Q = -2$, as well as ξ_ν. After EWSB the fermions with equal charges are mixed by the Higgs VEV.

Embedding 3
Similar to embedding 2, but:

$$\xi^e = (\mathbf{1}, \mathbf{3})_{-1} = \left[\xi^e_L = \begin{bmatrix} \nu'_L(+-) \\ e''_L(--) \\ \chi'_L(+-) \end{bmatrix} \quad \xi^e_R = \begin{bmatrix} \nu'_R(-+) \\ e_R(++) \\ \chi'_R(-+) \end{bmatrix} \right]. \tag{13}$$

Embedding 4   In this case the 5D leptons are:

$$\xi^l = (\mathbf{2}, \mathbf{2})_0 = \begin{bmatrix} L'_L(-+) & L'_R(+-) \\ L_L(++) & L_R(--) \end{bmatrix}; \tag{14}$$

$$\xi^\nu = (\mathbf{1}, \mathbf{3})_0 = \left[\xi^\nu_L = \begin{bmatrix} \chi_L(+-) \\ \nu''_L(--) \\ e'_L(+-) \end{bmatrix} \quad \xi^\nu_R = \begin{bmatrix} \chi_R(-+) \\ \nu_R(++) \\ e'_R(-+) \end{bmatrix} \right]; \tag{15}$$

$$\xi^e = (\mathbf{1}, \mathbf{3})_0 = \left[\xi^e_L = \begin{bmatrix} \chi'_L(+-) \\ \nu'_L(+-) \\ e''_L(--) \end{bmatrix} \quad \xi^e_R = \begin{bmatrix} \chi'_R(-+) \\ \nu'_R(-+) \\ e_R(++) \end{bmatrix} \right], \tag{16}$$

where L' and L are the $SU(2)_L$ doublets with $T^{3R} = \pm 1/2$ respectively, contained in ξ^l.

Embedding 5
Similar to embedding 4, but now ξ^ν is a 5D singlet.

$$\xi^\nu = (\mathbf{1}, \mathbf{1})_0 = \left[\nu''_L(--) \quad \nu_R(++) \right]. \tag{17}$$

Unlike for the previous four embeddings, in this case it is possible to have Majorana mass terms in locations other than the UV boundary, since ξ^ν is a gauge singlet. In particular, it is possible to write a Majorana mass term in the IR, M_{IR}. This will result in an order-one splitting of the masses of the Majorana components of the fourth-generation zero-mode neutrinos. As a result, in this embedding the fourth-generation zero-mode states are Majorana neutrinos, just as the first three-generation neutrinos. We will consider their spectrum in more detail below.

2.2 Charged leptons

Charged lepton masses are determined by the overlap of the left-handed and right-handed zero modes with the IR-localized Higgs. Their mass matrix is given by

$$M_{LR}^{e,ij} = m_1 \lambda_e^{ij} f(c_l^i) f(-c_e^j) \alpha(c_l^i, -c_e^j) , \tag{18}$$

$$f(c) = \left[\frac{1 - 2c}{1 - x^{1-2c}} \right]^{1/2} , \quad x = e^{-kL} , \tag{19}$$

where the the functions α and m_1 describe the Higgs localization and the Higgs VEV, respectively. For instance, if the Higgs arises from the condensation of the zero modes of the fourth-generation quarks, m_1 and α are defined by [3]

$$m_1 = \frac{\langle \bar{u}_L^{4(0)} u_R^{4(0)} \rangle}{k^2 x^2} \left(\frac{k}{M_P} \right)^3 , \tag{20}$$

$$\alpha(c_i, -c_j) = \frac{f(c_q^4) f(-c_u^4)(1 - x^{4 - c_q + c_u - c_i + c_j})}{(4 - c_q + c_u - c_i + c_j)} . \tag{21}$$

On the other hand, if the Higgs arises from the zero-mode of a fundamental 5D scalar field we have

$$m_1 = \langle H \rangle , \tag{22}$$

$$\alpha(c_i, -c_j) = \frac{(1 - x^{2 + \beta - c_i + c_j})\sqrt{2(1 + \beta)}}{(2 + \beta - c_i + c_j)(1 - x^{2 + 2\beta})^{1/2}} , \tag{23}$$

with β a function of the the 5D Higgs mass: $\beta = \sqrt{4 + m_H^2 L^2}$, (see for example Ref. [13]).

The masses of the charged leptons depend on the localization of the right-handed component, e_R^i, that is controlled by the bulk mass parameter c_e^i. The left-handed components of the light generations are almost delocalized, in such a way that we can obtain a light electron (a heavy τ^-) by localizing its right-handed component towards the UV (IR). To obtain a heavy fourth generation lepton both chiralities of e^4 must be localized towards the IR. The charged-lepton mass matrix M_{LR}^e is diagonalized as usual, by left and right unitary transformations $A_{L,R}^e$, such that the diagonal mass matrix is

$$M_D^e = U^{L\dagger} M_{LR}^e U^R . \tag{24}$$

The size of the mixings among the zero modes determines the size of the flavor-violating processes. Using Eq. (18) we expect the mixings to be of order [14, 15, 16]

$$U_{ij}^L \sim \frac{\min[f(c_l^i), f(c_l^j)]}{\max[f(c_l^i), f(c_l^j)]} , \qquad U_{ij}^R \sim \frac{\min[f(-c_e^i), f(-c_e^j)]}{\max[f(-c_e^i), f(-c_e^j)]} , \tag{25}$$

In the scenario with $c_l^i \simeq c_l$, since the Yukawas are all of the same order, there are no suppression factors in the mixings between the left-handed components of the light generations:

$$U_{ij}^L \sim \mathcal{O}(1) , \qquad i, j = 1, 2, 3. \tag{26}$$

In fact, we have enforced this result by adjusting the localization of left-handed components of the light generations. On the other hand, using Eq. (18) and $f(c_l^i) \simeq f(c_l^j)$, we expect the mixings between the right-handed components of the light generations to be of order

$$U_{ij}^R \sim \frac{\min[m_i, m_j]}{\max[m_i, m_j]} , \qquad m_{i,j} = m_e, m_\mu, m_\tau. \tag{27}$$

Thus we obtain a hierarchical U^R with small mixings between the light generations.

Since both the left- and right-handed components of e^4 are localized towards the IR, the mixings between the light generations and e^4 are of order:

$$U_{i4}^L \sim \frac{f(c_l^i)}{f(c_l^4)} , \qquad U_{i4}^R \sim \frac{f(-c_e^i)}{f(-c_e^4)} , \qquad i = 1, 2, 3. \tag{28}$$

In our model the τ_R has to be almost delocalized in order to obtain the correct value of m_τ. Therefore, the mixing between τ_R and e_R^4 can be rather large, as can be seen from Eq. (28). In our numerical scan (see next sections) we obtained

$$U_{\tau 4}^L \sim \frac{1}{2}\sqrt{\frac{m_\tau}{m_{e^4}}} , \qquad U_{\tau 4}^R \sim 2\sqrt{\frac{m_\tau}{m_{e^4}}} . \tag{29}$$

The large mixing $U_{\tau 4}^R$ arises as a consequence of the rather large τ-mass and the partial UV localization of τ_L. This result has important phenomenological consequences for e^4 FCNC decays, as we will discuss in Section 4.2.

2.3   Neutrinos

In what follows we consider the neutrino spectrum for embeddings 1 to 4. The case of embedding 5 will be consdered separately. There are two mass terms for the neutrinos: a Majorana mass for the right-handed components, which will affect almost exclusively the ones localized in the UV; and a Dirac mass determined by the overlap with the IR-localized Higgs, as seen in Eq. (5). The Dirac mass is given by Eq. (18), changing the index $e \to \nu$. On the other hand, the effective Majorana mass matrix for the zero-mode right-handed neutrinos can be written as

$$M_R^{\nu, ij} \simeq O(1) \, k \, F(-c_i) F(-c_j) , \quad F(c) = \left[\frac{2c - 1}{1 - x^{2c-1}} \right]^{1/2} . \tag{30}$$

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The spectrum of the light SM neutrinos is generated by the usual see-saw mechanism. As discussed above, the left-handed components of the first three generations are almost delocalized. The right-handed components have bulk mass parameters in the range

$$-0.4 \lesssim c_\nu^i \lesssim -0.28 \qquad i = 1, 2, 3 .$$ (31)

leading also to wave functions in the bulk, although somewhat more localized towards the UV.

Since we want a heavy fourth generation neutrino, we have to localize ν_R^4 towards the IR, in order to avoid the effect of the UV-localized Majorana mass, which would result in an unwanted see-saw and a light ν^4. Due to the exponential localization of the right-handed neutrino, the mass of ν^4 has a strong dependence on c_ν^4 (the left-handed component is localized in the IR to obtain a heavy e^4). In Fig. 1 we show m_{ν^4} as a function of c_ν^4, for different values of $c_l^4 = -0.4, 0, 0.4$. We have fixed $M_{UV} = 0.3 \, k$, all the Yukawas are equal to unity, and we have neglected the mixings between generations for simplicity. [1] For $c_\nu^4 \lesssim 0$, ν_R^4 has a sizable overlap with the UV and the see-saw mechanism becomes efficient, resulting in two Majorana neutrinos: one that is very light, and another one very heavy. For $c_\nu^4 \gtrsim 0$ the Majorana term becomes small enough and the Dirac mass dominates, resulting in both Majorana neutrinos being almost degenerate, or effectively in a Dirac fermion ν^4.

The right-handed neutrinos of the first three generations $\nu_R^{e,\mu,\tau}$, being localized towards the UV, have a very large effective Majorana mass ($\sim 10^{-5} M_{Pl}$, depending on c_ν). For this reason we integrate them out using the tree level equations of motion. The effective theory contains five Majorana neutrinos, three very light that reproduce the SM spectrum, and two almost degenerate heavy neutrinos which constitute an effective Dirac fourth-generation neutrino, as shown in Figure 1. The mass term of the effective theory can be written as

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2} \bar{N} M_{\text{eff}} N^c + \text{h.c.}$$ (32)

where N is defined by

$$N^t = (\nu_L^e, \nu_L^\mu, \nu_L^\tau, \nu_L^4, \nu_R^{4\,c}) ,$$ (33)

and M_{eff} is the effective mass matrix, that can be written in terms of the elements of M_{RR}^ν and M_{LR}^ν defined in Eq. (5), as shown in Appendix A. The neutrino spectrum is given by the

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[1] For a heavy neutrino, the mixings are very small, thus we do not expect important corrections in Fig. 1 from their effects.
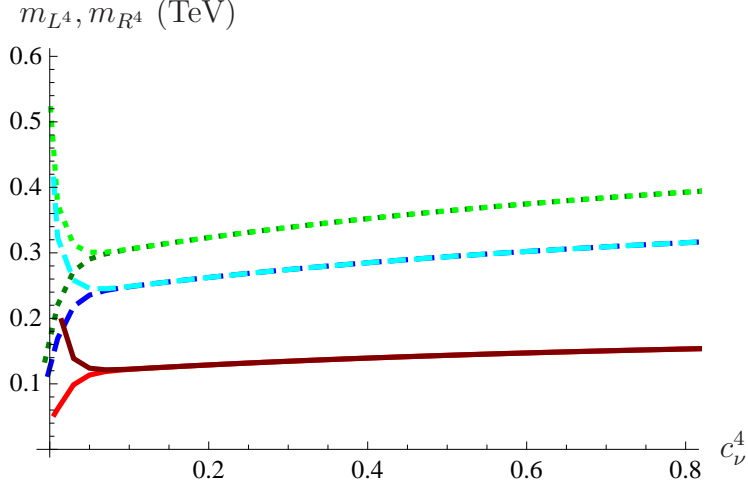
Figure 1: *Mass of the heavy fourth generation Majorana neutrinos, as a function of c_ν^4. The different lines correspond to different values of $c_l^4 = -0.4, 0, 0.4$. For c_ν^4 fixed, the larger m_{ν^4} corresponds to the smaller c_l^4, since in this case ν_L^4 is more localized in the IR. As long as ν_R^4 is near the UV, the Majorana mass becomes large, and the see-saw mechanism starts working, splitting the neutrinos in two: one very light and another very heavy, this happens for $c_\nu^4 \lesssim 0$.*

eigenvalues of M_{eff} and the neutrino mass-eigenstates by the eigenvectors of M_{eff}. Thus we diagonalize M_{eff} with an orthogonal transformation U^ν defined by

$$M_D^\nu = U^{\nu t} M_{\text{eff}} U^\nu \ . \tag{34}$$

Expanding in inverse powers of the large Majorana mass $M_R^{\nu,ij}$, with $i, j = 1, 2, 3$, the eigenvalues and eigenvectors corresponding to the heavy Majorana neutrinos, with masses $\sim \mathcal{O}(300)$ GeV, are given by:

$$m_{4,5}^\nu \;=\; \Big(\sum_{i=1}^4 M_{LR}^{\nu,i4} M_{RL}^{\nu,4i}\Big)^{1/2} \ , \tag{35}$$

$$\tilde{N}_{4,5} \;=\; (M_{LR}^{\nu,14}, M_{LR}^{\nu,24}, M_{LR}^{\nu,34}, M_{LR}^{\nu,44}, \pm m_{4,5}^\nu)\frac{1}{m_{4,5}^\nu \sqrt{2}} \ , \tag{36}$$

Thus, at leading order the heavy Majorana neutrinos $\tilde{N}_4$ and $\tilde{N}_5$ are degenerate, as advertised earlier and shown in Figure 1. It is possible to obtain similar expressions for the light eigenvectors and eigenvalues. We show these results in Appendix A.

The wealth of neutrino data collected in the last decade results in important constraints on these models through the precise measurements of neutrino mass differences and mixing angles.

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We now explore the possibility of reproducing these data in the model described above, with anarchic Yukawa couplings [2]. In the present scenario, considering only zero-mode fermions and integrating out the heavy right-handed neutrinos of the first three generations, we are left with four charged leptons and five Majorana neutrinos, as mentioned earlier. Then the charged current interactions in the physical basis are given by

$$\mathcal{L}_{CC} = \sum_{\alpha,a} \sum_n \frac{g_n}{\sqrt{2}} W_\mu^{+(n)} \bar{\nu}_L^a \gamma^\mu V_{a\alpha} e_L^\alpha + \text{h.c.} \ , \tag{37}$$

where $W^{(n)}$ and g_n are the KK W's and their couplings respectively, with $g_0 = g$ and g_n/g_0 depending on the localization of the zero-mode leptons: for the light (heavy) generations $g_1/g_0 \sim \mathcal{O}(0.1)$ ($\sim \sqrt{2k\pi r} \sim \mathcal{O}(10)$). V is the 5×4 mixing matrix that can be expressed in terms of the neutral and charged rotation matrices U^ν and U^L:

$$V_{a\alpha} = \sum_{i=1}^4 (U^{\nu t})_{ai} U_{i\alpha}^L \ , \qquad a = 1, \ldots 5 \ , \quad \alpha = 1, \ldots 4 \ . \tag{38}$$

The rotation matrix $U_{5\times5}^\nu$ is orthonormal, whereas $U_{4\times4}^L$ is unitary. The mixing matrix V satisfies the following constraints

$$(V^\dagger V)_{\alpha\beta} = \delta_{\alpha\beta} \ , \qquad (V\,V^\dagger)_{ab} = \delta_{ab} - (U^{\nu t})_{a5} U_{5b}^\nu \ . \tag{39}$$

Using the results of Eqs. (35) and (36) one can show that, to leading order, Eq. (39) reduces to:

$$(V\,V^\dagger) = \begin{bmatrix} 1_{3\times3} & 0 \\ 0 & \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \end{bmatrix} \tag{40}$$

If we consider only the first three generations, we can impose that V reproduces the observed neutrino mixing matrix. On the other hand, the mixings between zero-modes of the fourth generation and the light leptons of the first three generations are mostly determined by the localization of the left-handed wave functions. They are experimentally constrained, mostly from the decay $\mu^- \to e^-\gamma$, which receives new contributions induced by the presence of the new neutrinos running in the loop, as well as by the lack of unitarity of the 3×3 mixing matrix.

---

[2] However, we want to stress that this is not the full anarchic approach, since we have imposed a partial global flavor symmetry relating some of the 5D leptonic masses, as shown in Eq. (6). Had we considered anarchic 5D masses, we would have obtained hierarchical mixing, in contradiction with observation.

The contribution of a neutrino, with Dirac and right-handed Majorana masses, to the decay $\mu^- \to e^- \gamma$ can be written as [17]

$$
\begin{align}
T_i &= V_{\mu i}^{\dagger} V_{ie} H(m_i^2 / m_W^2) \tag{41} \\
H(x) &= \frac{1}{3(x-1)^4}(10 - 43x + 78x^2 - 49x^3 + 4x^4 + 18x^3 \log x) \ , \tag{42}
\end{align}
$$

leading to a branching fraction

$$
BR(\mu \to e\gamma) = \frac{\Gamma[\mu \to e\gamma]}{\Gamma[\mu \to e\nu\bar{\nu}]} = \frac{3\alpha}{32\pi} |\sum_i T_i|^2 \ . \tag{43}
$$

In the present model we have to sum over $i = 1, \ldots 5$, with $m_i \ll m_w$ for $i = 1, \ldots 3$. We obtain

$$
\sum_{i=1}^{5} T_i \simeq \sum_{i=1,23} V_{\mu i}^{\dagger} V_{ie} \left[H(0) + H'(0) \frac{m_i^2}{m_W^2} \right] + \sum_{i=4,5} V_{\mu i}^{\dagger} V_{ie} H\left(\frac{m_i^2}{m_W^2}\right) \ . \tag{44}
$$

In the SM only the first sum is present, and the GIM mechanism cancels the first term in the square brackets: $\sum_{i=1}^{3} V_{\mu i}^{\dagger} V_{ie} = 0$. In the present case we have instead

$$
\sum_{i=1,23} V_{\mu i}^{\dagger} V_{ie} = - \sum_{i=4,5} V_{\mu i}^{\dagger} V_{ie} \ . \tag{45}
$$

which leads to

$$
\sum_{i=1}^{5} T_i \simeq \sum_{i=4,5} V_{\mu i}^{\dagger} V_{ie} \left[H\left(\frac{m_i^2}{m_W^2}\right) - H(0) \right] + \ldots \ , \tag{46}
$$

where the dots stand for a SM type contribution, which can be neglected since is very small compared to the current bounds. Inserting Eq. (46) into (43) we obtain an upper bound for $|V_{4\alpha}|$ and $|V_{5\alpha}|$ in the present model. Taking as a reference value [1] $BR(\mu^- \to e^- \gamma) \lesssim 10^{-11}$ we obtain

$$
V_{\mu,i} \sim V_{e,i} \lesssim 10^{-2}, \qquad i = 4, 5. \tag{47}
$$

In the next section we will consider additional constraints from FCNC effects.

2.4 Model Parameter Scan

We have scanned over the model parameter space in order to find solutions compatible with all the available constraints. In our scan we have considered the following range for the 5D parameters:

- 5D Yukawa couplings of order $\mathcal{O}(1)$: $-2 \leq \lambda_{e,\nu,R}^{ij} \leq 2$, assuming real Yukawas for simplicity,

- 5D bulk mass parameters:

$$0.6 \leq c_l \leq 0.65, \quad -0.4 \leq c_\nu \leq -0.28, \quad 0.4 \leq c_e^i \leq 0.72$$
$$-0.5 < c_l^4 < 0.5, \quad 0 \leq c_\nu^4 < 0.5, \quad -0.5 < c_e^4 < 0.5$$

Finally, we considered $k\,e^{-kL} = 1$ TeV (except in the cases where we explicitly mention a larger KK scale), and assumed a Higgs arising from the condensation of the quarks of the fourth generation, see Eqs. (20) and (21), with $c_q^4 < 0$ and $c_u^4 > 0$. We observe that in this region of parameter space, it is possible to obtain lepton masses and mixings between the light states compatible with experiment. The same solutions result in fourth generation-leptons heavy enough to have evaded direct detection bounds. We have also studied the amount of tuning in the space $\{c_l^i, c_\nu^i\}$, $i = 1, 2, 3$, needed to obtain the right pattern of mixings. We have allowed a small random variation of the 5D masses in our numerical scan: $\{c_l^i = c_l + \Delta_l^i, c_\nu^i = c_\nu + \Delta_\nu^i\}$. Our results show that, in order to satisfy the constraints, Δ can not be larger than $\Delta_l^i \sim \Delta_\nu^i \sim 0.01$. As a benchmark point in the parameter space of 5D masses we consider

$$
\begin{aligned}
c_l &= 0.59\,, & c_\nu &= -0.34\,, & c_e &= -0.73\,, & c_\mu &= -0.61\,, & c_\tau &= -0.52\,, \\
c_l^4 &= 0\,, & c_\nu^4 &= 0.30\,, & c_e^4 &= 0.35\,.
\end{aligned}
\tag{48}
$$

In Appendix B we show an example of a specific solution satisfying all the constraints.

2.5 Neutrino Spectrum in Embedding 5

We discuss here the spectrum of zero-mode neutrinos in Embedding 5, presented in Section 2.1. In it the right-handed neutrino zero-modes come from ξ^ν which transforms as $(\mathbf{1}, \mathbf{1})_0$ under $SU(2)_L \times SU(2)_R \times U(1)_X$. This choice allows the presence of a Majorana mass term in the IR brane

$$\mathcal{S}_{IR} = \int dx^4 dy \sqrt{g} \left(-\frac{1}{2} \bar{\xi}_R^{\nu c} \mathcal{M}_{IR} \xi_R^\nu + \text{h.c.} \right) \frac{\delta(y - L)}{\Lambda_{UV}}\,,
\tag{49}$$

which induces a Majorana mass

$$M_{IR}^{ij} = O(1)\,(ke^{-kL})\,f(-c_i)f(-c_j)
\tag{50}$$

13

for the zero-mode neutrinos. Since the first three-generation neutrinos are UV-localized they will not be significanty affected by M_{IR}. Thus the effect of M_{IR} can be schematically described by

$$\mathcal{L} = -\frac{1}{2}\,\bar{\nu}_R^4 M_{IR}\nu_R^{4c} - \bar{\nu}_L^4 M_{LR}^4 \nu_R^4 + h.c. \tag{51}$$

where M_{LR}^4 is the zero-mode Dirac mass for the fourth-generation neutrino, and we have neglected mixing with the lighter generations. This leads to a splitting between the two Majorana states induced by M_{IR}, which is typically of order one. The light state is an admixture of ν_L^4 and ν_R^4, but it has typically a larger fraction of ν_L^4 as long as $M_{IR} > M_{LR}^4$. The lighter state could be rather light given that there is a mild see-saw controlled by $\mathcal{M}_{IR}$. For instance, it is quite natural in this embedding to consider a lighter Majorana states with masses as small as 100 GeV.

3 Constraints from Flavor Violation

In this section we study the constraints on these models from lepton flavor violation. The bounds from the dipole operator inducing $\mu^- \to e^-\gamma$ were shown in the previous section. Here we concentrate on flavor violating decays of the charged leptons, such as $\mu^- \to e^- e^+ e^-$ and $\mu^- - e^-$ conversion in nuclei. As we show below, there is also a significant contribution to the decay of e^4 through FCNC, resulting in $e^4 \to \tau^- f\bar{f}$, with f a SM light fermion.

We consider the effects from the interactions between the would-be zero-mode leptons and the neutral vector bosons, neglecting the effects from Higgs boson exchange since they are suppressed by the small SM lepton masses[3]. Although the couplings between the KK vector bosons and the leptons are not universal due to the generation-dependent localization of the leptons, the neutral-current interactions are diagonal in the flavor basis before electroweak symmetry breaking (EWSB)

$$\mathcal{L}_{NC} = \sum_a \sum_n Z_\mu^{(n)} \bar{e}^a \gamma^\mu (g_{na}^L P_L + g_{na}^R P_R) e^a \ , \tag{52}$$

where $g_{0a}^{L,R} = g_{SM}^{L,R}$ is the usual SM Z-coupling. After EWSB, the Z boson arises primarily from the mixing between the zero and the first KK neutral modes, $Z^{(0)}$ and $Z^{(1)}$ (we neglect the mixing with the heavier KK-modes for simplicity in this analysis). Diagonalizing the vector

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[3]See Refs. [14, 15, 16, 19] for FCNC effects arising from the Higgs in composite Higgs models.

boson mass matrix, the physical Z boson at leading order is given by

$$Z = Z^{(0)} - f\frac{m_Z^2}{m_{KK}^2}Z^{(1)} , \qquad (53)$$

where $f \sim \sqrt{2\,kL} \sim \mathcal{O}(10)$ is a factor parametrizing the mixing, and in general has a mild dependence on the Higgs localization. Thus, since the Z boson has a sizable projection over the first KK mode, the neutral lepton interactions are not universal after EWSB. In order to obtain the neutral interactions for the mass eigenstates we rotate the flavor basis in (52), which results in

$$\mathcal{L}_{NC} = \sum_{a,b}\left[Z_\mu \bar{e}^a \gamma^\mu (g_{ab}^L P_L + g_{ab}^R P_R)e^b + \sum_n Z_\mu^n \bar{e}^a \gamma^\mu (g_{nab}^L P_L + g_{nab}^R P_R)e^b \right] , \qquad (54)$$

where Z^n stands for the heavy neutral vectors arising from the mixed KK modes. The flavor-violating Z couplings in the mass eigenbasis are given by

$$g_{ab}^{L,R} = -f\frac{m_Z^2}{m_{KK}^2}(U^{L,R\dagger}G^{L,R}U^{L,R})_{ab} , \qquad G^{L,R} = \mathrm{diag}(g_{1e}^{L,R}, g_{1\mu}^{L,R}, g_{1\tau}^{L,R}, g_{14}^{L,R}) , \qquad (55)$$

where $g_{1a}^{L,R}$ are the diagonal flavor dependent couplings with $Z^{(1)}$, defined in Eq. (52).

The flavor-violating couplings relevant for the $\mu^- - e^-$ transitions are determined by

$$\begin{aligned}(U^{L,R\dagger}G^{L,R}U^{L,R})_{e\mu} = \quad & U_{12}^{L,R\dagger}(g_{1\mu}^{L,R} - g_{1e}^{L,R})U_{22}^{L,R} + U_{13}^{L,R\dagger}(g_{1\tau}^{L,R} - g_{1e}^{L,R})U_{32}^{L,R} + \\ & U_{14}^{L,R\dagger}(g_{1e^4}^{L,R} - g_{1e}^{L,R})U_{42}^{L,R} , \end{aligned} \qquad (56)$$

where we have used the unitarity of the rotation matrices. There are similar expressions for the other flavor-violating neutral interactions $(U^{L,R\dagger}G^{L,R}U^{L,R})_{\alpha\beta}$, $\alpha \neq \beta$.

Using the estimates of Section 2.2 we can obtain the size of the flavor-violating couplings. From (56) we see that for the choice $c_l^i = c_l$, $i = 1, 2, 3$, the couplings g_{1i}^L have the same value for the light generations and the only contributions to flavor-violating processes in the left-handed sector are due to the fourth generation. Allowing for small departures from universal localization, $c_l^i = c_l + \Delta_l^i$, we also obtain contributions from the light generations. Using the results from our numerical scan allowing $\Delta_l^i \sim 0.01$, the contributions to FCNC processes relevant for $\mu^- - e^-$ transitions come from the flavor-violating factors

$$U_{12}^{L\dagger}(g_{1\mu}^L - g_{1e}^L)U_{22}^L \quad \sim \quad \frac{1}{2} \times 10^{-3}g^L \times \frac{1}{2} \sim 10^{-4}g^L , \qquad (57)$$

$$U_{13}^{L\dagger}(g_{1\tau}^L - g_{1e}^L)U_{32}^L \quad \sim \quad \frac{1}{2} \times 10^{-3}g^L \times \frac{1}{2} \sim 10^{-4}g^L , \qquad (58)$$

$$U_{14}^{L\dagger}(g_{1e^4}^L - g_{1e}^L)U_{42}^L \quad \sim \quad 10^{-2} \times 5g^L \times 10^{-2} \sim 5 \times 10^{-4}g^L . \qquad (59)$$

In (59) we have considered the fact that e_L^4 is not completely localized in the IR, and for this reason the coupling with the KK vector is somewhat smaller than f, $g_{14}^L \sim f/2 \sim 5$. On the other hand, since the light generations are localized towards the UV, their couplings to the KK vectors are suppressed by: $g_{1i}^L \sim 1/f \sim \mathcal{O}(0.1)$ for $i = 1, 2, 3$. The dominance of the e^4 contribution suggested by Eqs. (57-59) is confirmed in our more detailed numerical studies.

Similarly, we can obtain contributions of the right-handed sector, given by

$$U_{12}^{R\dagger}(g_{1\mu}^R - g_{1e}^R)U_{22}^R \sim \frac{m_e}{m_\mu} \times 10^{-3}g^R \times 1 \sim 10^{-5}g^R \ , \tag{60}$$

$$U_{13}^{R\dagger}(g_{1\tau}^R - g_{1e}^R)U_{32}^R \sim \frac{m_e}{m_\tau} \times 0.1g^R \times \frac{m_\mu}{m_\tau} \sim 10^{-5}g^R \ , \tag{61}$$

$$U_{14}^{R\dagger}(g_{1e^4}^R - g_{1e}^R)U_{42}^R \sim \frac{m_e}{m_{e^4}} \times 5g^R \times \frac{m_\mu}{m_{e^4}} \sim 5 \times 10^{-5}g^R \ . \tag{62}$$

Again e^4 gives the dominant contribution. We also note that the flavor-violation effect in the left-handed sector is an order of magnitude larger than the one coming from the right-handed sector. Taking into account all $\mathcal{O}(1)$ coefficients, a numerical scan results in $(U^{R\dagger}G^R U^R)_{e\mu} \sim 1/3 \times 10^{-4}$.

The most stringent constraints come from the upper limits for $\mu^- \to e^- e^+ e^-$ decay branching ratio, and from the $\mu^- - e^-$ conversion rate in nuclei. Following the analysis of [20] and [21], we define the relevant effective couplings by

$$-\mathcal{L}_{eff} = \frac{4G_F}{\sqrt{2}} \quad [\ g_3(\bar{e}_R\gamma_\mu\mu_R)(\bar{e}_R\gamma_\mu e_R) + g_4(\bar{e}_L\gamma_\mu\mu_L)(\bar{e}_L\gamma_\mu e_L)$$
$$+ \ g_5(\bar{e}_R\gamma_\mu\mu_R)(\bar{e}_L\gamma_\mu e_L) + g_6(\bar{e}_L\gamma_\mu\mu_L)(\bar{e}_R\gamma_\mu e_R) \ ] + \text{h.c.} \ . \tag{63}$$

with G_F the Fermi constant. Normalizing to the $\mu^- \to e^- \bar{\nu}\nu$ branching ratio, we can write the branching ratio for $\mu^- \to e^- e^+ e^-$ as

$$\frac{BR(\mu^- \to e^- e^+ e^-)}{BR(\mu^- \to e^- \bar{\nu}\nu)} = 2(|g_3|^2 + |g_4|^2) + |g_5|^2 + |g_6|^2 \ . \tag{64}$$

The current experimental limit is $BR(\mu^- \to e^- e^+ e^-) < 10^{-12}$ [1]. The $\mu^- - e^-$ conversion rate in nuclei is given by [21, 22]:

$$B_{\text{conv}}(\mu - e) = \frac{2p_e E_e G_F^2 m_\mu^2 \alpha^3 Z_{eff}^2 Q_N^2}{\pi^2 Z \Gamma_{\text{capt}}}(|g_{e\mu}^L|^2 + |g_{e\mu}^R|^2) \ , \tag{65}$$

where $g_{e\mu}^{L,R}$ are the flavor-violating Z couplings defined in (55) and α is the fine structure constant. The other factors depend on the specific nuclei involved in the reaction and can be

found in Ref. [22]. The most constraining limits arise from the SINDRUM II experiment at PSI using Au. The corresponding bound is $B_{\text{conv}}(\mu^- - e^-) < 0.7 \times 10^{-12}$, at 90% C.L. [1].

In order to obtain the model predictions for the conversion rate, we consider both the contributions from the Z as well as from the neutral KK vectors in (54). From (55) the dominant contributions to the effective couplings g_{3-6} are given by

$$g_3 \simeq \frac{g^R}{m_{KK^2}} \left(f - \frac{g_{1e}^R}{g^R} \right) (U^{R\dagger} G^R U^R)_{e\mu}, \qquad g_4 \simeq \frac{g^L}{m_{KK^2}} \left(f - \frac{g_{1e}^L}{g^L} \right) (U^{L\dagger} G^L U^L)_{e\mu},$$

$$g_5 \simeq \frac{g^L}{m_{KK^2}} \left(f - \frac{g_{1e}^L}{g^L} \right) (U^{R\dagger} G^R U^R)_{e\mu}, \qquad g_6 \simeq \frac{g^R}{m_{KK^2}} \left(f - \frac{g_{1e}^R}{g^R} \right) (U^{L\dagger} G^L U^L)_{e\mu}, \qquad (66)$$

where, in each case, the first term comes from Z exchange and the second one is from direct KK exchange. The latter can be neglected since $f \gg g_{1e}^{L,R}/g^{L,R}$, just as for the case with three generations [21].

The experimental bounds can now be used constrain m_{KK}. Making use of the naive estimates of Eqs. (57-62) and the results of (66) in Eqs. (64) and (65), we obtain:

$$BR(\mu^- \to e^- e^+ e^-): \ m_{KK} \gtrsim 4 \text{ TeV}, \qquad B(\mu^- - e^-)_{\text{conv.}}: \ m_{KK} \gtrsim 6 \text{ TeV}. \qquad (67)$$

A more precise statement about the experimental constraints on the KK scale can be obtained scanning over the parameter space defined in Section 2.3, using the benchmark point of Eq. (48). The results are in Figure 2 (a), where we show the predictions of the model for $\mu^- \to e^+ e^- e^-$ and $\mu^- - e^-$ conversion in terms of the ratios to the experimental bounds, $R(\mu^- \to e^- e^+ e^-)$ and $R(\mu^- - e^-)_{conv.}$. The points in the plot correspond to different sets of anarchic 5D Yukawas that reproduce the observed spectrum and mixings and also satisfy the $\mu^- \to e^- \gamma$ bounds of (47). We have also allowed a small violation of universal localization for the left-handed light leptons, $\Delta_{l,\nu}^i \sim 0.01$.

The 90% C.L. region allowed by both experiments is the lower left corner. Note that for $m_{KK} = 2.4$ TeV (red points) almost no configuration satisfies the experimental constraints, whereas for $m_{KK} = 6$ TeV (black crosses) a sizable number of the solutions lie within the limits. The most stringent constraint comes from $\mu^- - e^-$ conversion. If we only consider the bound from $\mu^- \to e^- e^+ e^-$, a portion of the parameter space with $m_{KK} = 2.4$ TeV is allowed.

Figure (2) (a) points to $m_{KK} \gtrsim 6$ TeV, introducing a little hierarchy that renders the model somewhat unnatural. However, the naive choice of parameters resulting in Figure (2) (a) is not an optimal one. As noted above, the largest contributions arise from the left-handed flavor violating interactions, at least one order of magnitude larger than the right-handed ones. It is
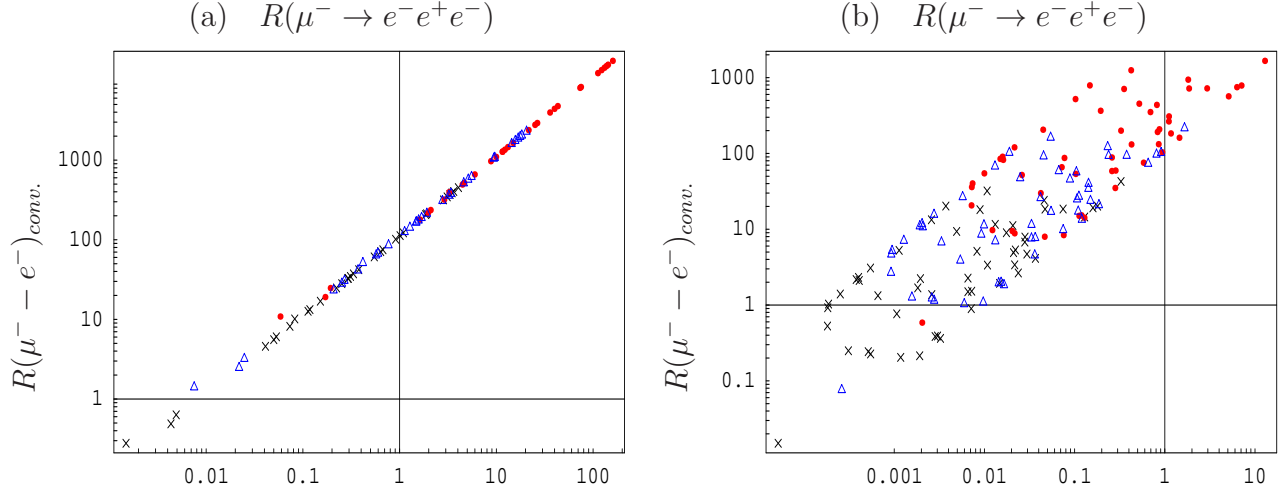
Figure 2: *Predictions for $\mu^- \to e^- e^+ e^-$ and $\mu^- e^-$ conversion normalized to the experimental limits. The points correspond to different sets of 5D parameters compatible with lepton flavor data and satisfying the constraints from $\mu \to e\gamma$ in (47). The dots (red), triangles (blue) and crosses (black) correspond to $m_{KK} = 2.4, 4, 6$ TeV. The horizontal and vertical lines are the experimental bounds. On the left we show the results for the region of the parameter space defined in Section 2.3. On the right we show the results when the localization is chosen so that left-handed and right-handed contributions are of the same order. The dispersion in this case is due to different sets of Yukawas $\sim \mathcal{O}(1)$ and to small variations in the 5D fermion masses: $\Delta^i \sim 0.01$.*

possible to decrease the amount of flavor violation in the left-handed sector by increasing the UV localization of the left-handed light leptons. For instance, if we consider $c_l \sim 0.65$, since $f(0.65)/f(0.6) \sim 1/5$, this would result in a suppression factor of $\sim 1/25$ in the contribution of e_L^4 to $g_{e\mu}^L$. Also, for larger c_l^i, the KK couplings g_{1i}^L become closer to the universal value (obtained for $c_l^i = \infty$), and the cancellation in $g_{1i}^L - g_{1e}^L$ becomes more efficient. Of course, in order to obtain the observed charged lepton masses, we have to increase the IR localization of the right-handed light leptons, increasing at the same time the size of the flavor violation in the right-handed sector. Roughly speaking we increase $g_{e\mu}^R$ by a factor equal to the one suppressing $g_{e\mu}^L$. Therefore, we reach the minimum when the flavor violation in the left and right-handed sectors are of the same order. By playing with the localization in this way, we can decrease (increase) $g_{e\mu}^L$ ($g_{e\mu}^R$) by a factor ~ 4, lowering the contribution to $\mu^- - e^-$ conversion by one order of magnitude. More specifically, we want the size of the left-handed and right-handed contributions to be of the same order

$$\frac{f(c_l^1)f(c_l^2)}{f(c_l^4)^2} \sim \frac{f(-c_e^1)f(-c_e^2)}{f(-c_e^4)^2} \ . \tag{68}$$

We can satisfy (68) and obtain the correct spectrum by choosing the 5D fermion masses

$$c_l^1 \simeq c_l^2 \simeq 0.625 \ , \qquad c_e \simeq -0.68 \ , \qquad c_\mu = -0.575 \ , \qquad (69)$$

and the usual localization for the fourth generation. For this configuration the contributions from mixings between the first two generations are as large as the one from the fourth generation. We estimate them to be

$$(U^{L\dagger}G^L U^L)_{e\mu} \sim (U^{R\dagger}G^R U^R)_{e\mu} \sim 10^{-4} \ . \qquad (70)$$

With these estimates we obtain:

$$BR(\mu^- \to e^- e^+ e^-): \ m_{KK} \gtrsim 2 \ \text{TeV} \ , \qquad B_{\text{conv}}(\mu^- - e^-): \ m_{KK} \gtrsim 4 \ \text{TeV} \ . \qquad (71)$$

In Fig. 2 (b) we show the results of the scan for this configuration. As expected, the constraints now admit solutions with lighter values of m_{KK}, as low as $m_{KK} \sim 2.4$ TeV.

In addition to optimal localization, there is another way to suppress lepton flavor violation. This can be achieved with certain choices of embedding of the lepton sector in the 5D gauge theory. Although up to now we have not addressed this point, we must make such choice in order to define the phenomenology of the model since the 5D gauge symmetry is $SU(2)_L \times SU(2)_R \times U(1)_X$, and not just the SM electroweak gauge sector. This FCNC-suppressing mechanism is independent of the localization of the zero modes. Instead, we will show in the next section that there is a symmetry that protects the Z interactions of the left-handed fermions [23, 24].

3.1 Custodial symmetry for flavor violating Z couplings

As promised in the previous section, choosing the embedding of the lepton sector in the 5D bulk gauge theory can significantly relax the lepton flavor-violation bounds, eliminating the need for a higher KK mass scale. By properly choosing the representation of the leptons under the 5D gauge symmetry, it is possible to protect the Z couplings from shifts generated by its mixing with the KK resonances [23]. It is generally the case that if the 5D couplings of the $SU(2)_{L,R}$ gauge groups are taken to be equal and the fermion involved satisfies $T^{3L} = T^{3R}$, there will be a P_{LR} custodial symmetry. In the present case, this symmetry can be realized by embedding the left-handed doublet in a $(\mathbf{2},\mathbf{2})_0$, protecting the left-handed coupling of the charged leptons, such as in Models 4 and 5 in the previous section. Note that we can not protect the ν_L simultaneously, because $T^{3L} = -T^{3R}$ for the ν_L in this case. Instead we would have

to choose a $(\mathbf{2}, \mathbf{2})_{-1}$ to protect the ν_L coupling. To protect the coupling of the right-handed fermions, it is possible to invoke also a P_C symmetry, realized when $T^{3L} = T^{3R} = 0$. Thus we can choose a $(\mathbf{1}, \mathbf{1})_{-1}$ or $(\mathbf{1}, \mathbf{3})_{-1}$ for e_R, as in Models 2 and 3, respectively. It is not possible to protect both, the left-handed and right-handed couplings of charged leptons at the same time, because they have different X charges.

We showed in the previous section that, for the region of the parameter space chosen in Section 2.3, it is necessary to decrease the flavor-violating effects for the *left-handed charged* leptons. This can be done in Embeddings 4 and 5. Therefore, the flavor-violating Z couplings in the left-handed sector are suppressed, and we can lower the KK scale (see Ref.[24] for related discussions). Although we have not made a detailed analysis, we expect that this choice of embeddings would alleviate the tension from FCNC, allowing for a considerably lower value of m_{KK}.

Notice that the contributions to FCNC arising from direct KK exchange (as opposed to the effects from mixing between $Z^{(0)}$ and $Z^{(n)}$) are still present. However, these effects are suppressed by a factor $\sim 1/f$, and we expect this contribution to be sub-dominant.

4   Phenomenology

Many of the lepton properties in this framework are independent of the embeddings on the higher dimensional gauge group, and for this reason they are robust predictions if there is a fourth generation of leptons in 5D. In the following we will discuss some of these generic properties, and then we will comment on some model-dependent properties.

4.1   Light KK Spectrum

An important prediction of the model is the presence of light KK leptons of the fourth generation (charged and neutral), lighter than the gauge KK modes [18]. This can be understood by the following reasons. The large 5D gauge symmetry (larger than the SM one) leads to 5D fermions transforming under the full gauge transformations. To avoid an excess of zero-modes some of the partners of e^4 and ν^4 have $(\pm, \mp)$ boundary conditions. For a 5D mass $-1/2 \lesssim c \lesssim 1/2$ the mass of the first KK mode of a right-handed fermion with $(-, +)$ boundary conditions can be approximated by:

$$m_{\mathrm{KK}} \simeq A \, k e^{-kL} \sqrt{\frac{1}{2} - c} \, , \tag{72}$$

20

where $A \sim 2$ is approximately constant. Thus, the mass of the first right-handed KK mode is parametrically smaller than the mass of a gauge KK mode, with the suppression given by $\sqrt{1/2 - c}$. For a left-handed fermion with $(-, +)$ boundary conditions a similar situation holds changing $c \to -c$.

As discussed in Section 2, in order to obtain heavy leptons for the fourth generation, the 5D masses have to be in the range: $-1/2 \lesssim c_{l,\nu,e}^4 \lesssim 1/2$. Moreover, since the see-saw mechanism has a strong dependence with c_ν, we need $c_\nu^4 \gtrsim 0$ to avoid a new light neutrino, experimentally excluded.

From the above arguments and the lepton embeddings of Section 2.1 we obtain that the first KK modes arising from the $\mathrm{SU}(2)_R$ partners of e^4 and ν^4 are light. This is a consequence of the large 5D gauge symmetry and the heaviness of the fourth generation[4]. In terms of the 4D dual description of the theory the light fermionic resonances arise as a consequence of the large global symmetry of the strong sector and the fourth generation being almost composite states.

We will consider the different embeddings of the previous section to study the range of masses of the light KK fermions. As we see below, the existence of these light states is a generic property present in all embeddings. Although we only show the results for Models 1 and 5, that have some important differences, we have explicitly checked that the range of masses is similar for all the embeddings of Section 2.1. We have checked in our calculations that the effects arising form generation mixing are not larger than 25%, thus we will neglect the mixings between generations for this analysis, and we will consider just the fourth generation that will lead to light KK fermions. For the purpose of this simplified discussion, we will consider that the Higgs is localized in the IR boundary with a δ-function.

4.1.1 Model 1

After EWSB, there are two towers of leptons: one tower of charged fermions with a light KK mode whose mass is controlled by c^ν (note that $e'_R(-+)$ has the appropriate boundary conditions), and one tower of neutral fermions with a light KK mode whose mass is controlled by c^e ($\nu''_R(-+)$ has the appropriate boundary conditions). The spectrum of charged fermions is given by

$$\text{zeroes} \left[\frac{f_{\alpha_l}^R}{f_{\alpha_l}^L} + (\mathcal{M}^e z_1)^2 \left(\frac{f_{\alpha_e}^R}{f_{\alpha_e}^L} + \frac{f_{\alpha_\nu}^R}{f_{\alpha_\nu}^L} \right) \right] |_{z_0} , \tag{73}$$

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[4]A similar effect is present in the quark sector for fields with zero-modes giving rise to heavy fermions.

where $f^{L,R}_{\alpha_\psi}$ is the KK wave function of the left (right)-handed charged fermion ψ and $\mathcal{M}^e$ is the Dirac mass from a localized IR Higgs. The spectrum of neutral fermions is given by a similar equation.

Figure 3 shows the mass of the lightest charged (neutral) KK mode as a function of c^ν (c^e), and the prediction given by Eq. (72) with $A = 2.3$. We have made a random scan over the 5D masses constraining $-0.5 \leq c^4_{l,e\nu} \leq 0.5$ and we have fixed $\mathcal{M}^{e,\nu} = 0.25\,k\,e^{-kL}$, that corresponds to Yukawas of order one. In Figure 4 we show the mass of the lightest charged KK versus
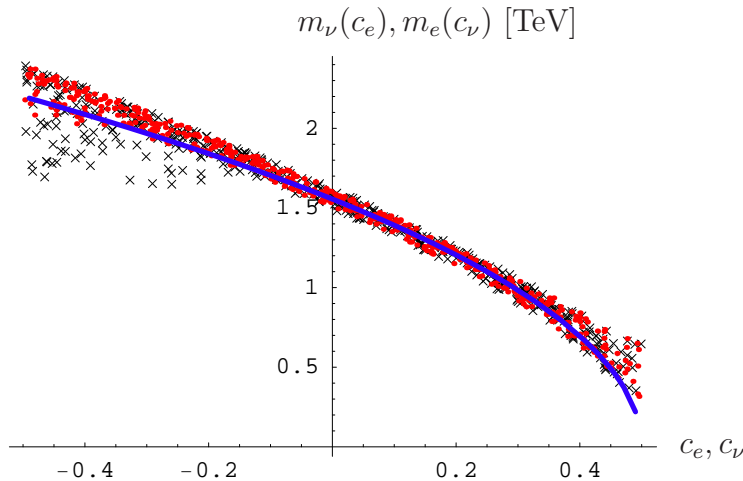

Figure 3: *Model 1. Red dots (black crosses) correspond to the mass of the lightest charged (neutral) KK fermion as a function of c_ν (c_e). The blue line is the prediction given by Eq. (72). We have made a random scan with $-0.5 \leq c^4_{l,e\nu} \leq 0.5$ and we have fixed $\mathcal{M}^{e,\nu} = 0.25\,k\,e^{-kL}$.*

the mass of the zero mode ν^4 using the same scan as in the previous plot. We can clearly see that, since a heavy ν^4 requires $c^\nu_4 \gtrsim 0$, this model predicts a charged KK not heavier than $\sim 1.5\,k\,e^{-kL}$ ($\simeq 1.5$ TeV for $k\,e^{-kL} \simeq 1$ TeV).

The new fermions give contributions to the oblique parameters. The most important constraint comes from the contributions of the would be zero modes of the fourth generation to the T parameter, that is constrained by electroweak precision measurements to be not much larger than $T \sim 0.1$[5]. Fixing the leptonic contribution to be $\Delta T = 0.1$ we constrain the isospin splitting to be $\Delta m = |m_{\nu^4} - m_{e^4}| \sim 74$ GeV. This result has important consequences for the phenomenology, since $\Delta m < m_W$ and then the leptons of the fourth generation will preferentially decay to $W + l$, with l labeling the SM leptons. However this result depends

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[5]The precise value of the bound on T depends on the value of the other electroweak observables in the model.
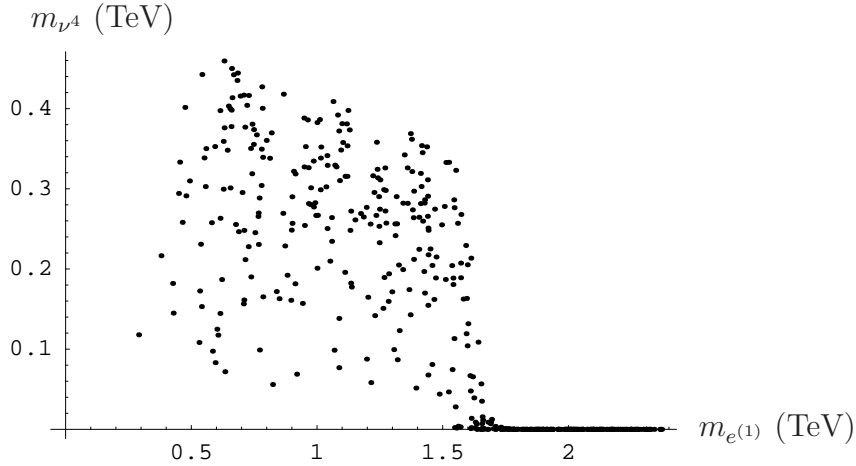
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Figure 4: *Model 1. Mass of the zero mode ν^4 as a function of the mass of the first KK charged fermion $e^{(1)}$. We have varied $-0.5 \leq c^4_{l,e,\nu} \leq 0.5$ and we have fixed $\mathcal{M}^{e,\nu} = 0.25/z_1$.*

on the precise value of ΔT. For instance, for $\Delta T \sim 0.3$ we obtain $\Delta m \sim 130$ GeV$> m_W$. Thus, electroweak precision constraints prefer a small isospin splitting, although we can not completely exclude the possibility $\Delta m > m_W$. The constraints on the T parameter exclude a region of the parameter space of the 5D model. Demanding $\Delta T = 0.1$ we keep $\sim 60\%$ of the parameter space in Model 1, where we have considered only the region of the parameter space where ν^4 and e^4 are heavy enough. Thus, the result has a mild dependence on this lower limit. For example, for $m_{\nu^4, e^4} > 50$ GeV (250 GeV) we keep 51% (72%) of the parameter space.

A similar analysis can be done for Models 2-4. In all these models we need $c^4_\nu \gtrsim 0$ to obtain a heavy ν^4, therefore light KK modes will arise from the $\mathrm{SU}(2)_R$ partners of ν_R, that is embedded in a triplet of $\mathrm{SU}(2)_R$. In Models 2 and 3 there is a light mode with the electron quantum numbers, $Q = -1$, and there is also an exotic state χ with $Q = -2$, whereas in Model 4 χ has $Q = +1$. Since the left-handed doublet is embedded in a bi-doublet of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, there are also light charged and exotic KK modes associated to the $\mathrm{SU}(2)_R$ partners of L_L, *i.e.*: L'_L. In this case their masses are controlled by c^4_l. In Models 3 and 4 the right-handed electron is embedded in a triplet of $\mathrm{SU}(2)_R$, thus there are also light states associated to the $\mathrm{SU}(2)_R$ partners of e^4_R, with the spectrum controlled by c^4_e. After the Higgs acquires a VEV, the fields with the same electric charge are mixed, thus the mass of the light states can depend on the parameters $c^4_{l,e,\nu}$, according to Eq. (72). Let us discuss briefly Model 2, as an example: there are light neutral sates for c^l near $-1/2$, and there are light charged and exotic states for c^l or $-c^\nu$ near $-1/2$ (the last condition is similar to Model 1). As discussed previously, to obtain a

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heavy fourth generation we need $-1/2 \lesssim c_{l,e}^4 \leq 1/2$ and $0 \lesssim c_\nu^4 \leq 1/2$, thus the last condition guarantees the existence of light charged and exotic states associated to the ν_R partners, and the other conditions can give an extra suppression in the mass of the first KK.

4.1.2 Model 5

We consider this model separately, due to its somewhat distinct features. In this case, ξ^ν is a singlet, implying that fermions with $(-,+)$ boundary conditions arise from ξ^l and ξ^e. As previously, to obtain a heavy e^4 we need $-1/2 \lesssim c_{1,e}^4 \lesssim 1/2$. Thus there is a region of the parameter space leading to light fermionic KK resonances ($-1/2 \lesssim c_l^4 \lesssim 0$ and/or $0 \lesssim c_e^4 \lesssim 1/2$), and another region $0 \lesssim c_l^4 \lesssim 1/2$ and $-1/2 \lesssim c_e^4 \lesssim 0$ where we can still obtain a heavy fourth generation and the suppression in the KK mass is not so large. Thus, once we select the region of the parameter space leading to heavy leptons, Model 5 will give, in general, light fermionic resonances, but there are some regions where the suppression is small (roughly 20% of the allowed region). In Figure 5 we plot the KK mass in terms of the masses of e^4 and ν^4, using $\mathcal{M}_{IR} = 0.25k$ for the IR-localized Majorana mass. Besides the usual charged and neutral states there is another state with $Q = +1$. After EWSB the mass eigenstates arise from mixing between the upper component of L' and χ in Eqs. (14) and (17).

Note that in this model there are no fields with $(-+)$ boundary conditions and $Q = -1$, thus there is no light KK associated to the electron, as can be seen in Figure 5. The contributions
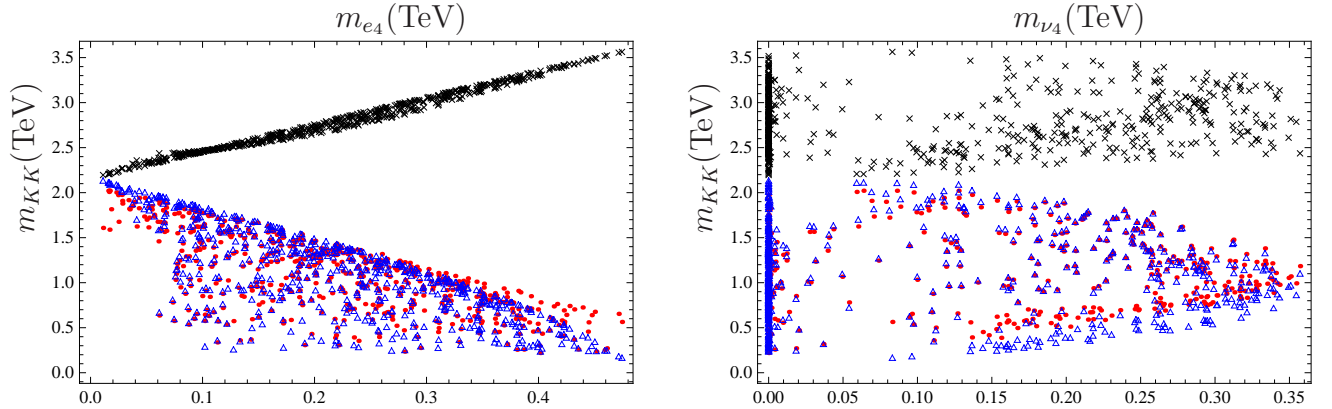

Figure 5: *Model 5. Crosses (black), dots (red) and triangles (blue) correspond respectively to the mass of the lightest charged, neutral, exotic KK fermions. On the left (right) we show the KK mass as a function of the mass of the zero-mode electron (neutrino) of the fourth generation e^4 (ν^4). We have made a random scan over the parameter space with $-0.5 \leq c_{l,e,\nu}^4 \leq 0.5$.*

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of the zero modes to the T parameter are similar to Model 1.

4.2 Couplings of Fourth Generation Leptons

In this section we study some features of the interactions involving fourth-generation leptons that have an impact on their collider phenomenology. Since the IR localization of the fourth generation is larger than the SM one, the following relation is satisfied: $M_{LR}^{\nu,44} \gg M_{LR}^{\nu,i4}$ for $i = 1, 2, 3$, see Eq. (36). Using this result, to leading order we can write U^ν as:

$$U^\nu \simeq \left[\begin{array}{cc} U_{3\times3}^\nu & 0_{3\times2} \\ 0_{2\times3} & \left[\begin{array}{cc} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{array} \right] \end{array} \right] . \tag{74}$$

Inserting Eq. (74) in (37) we obtain, at this level of approximation:

$$\mathcal{L}_{CC} \supset \sum_{\alpha=1,2,3} \frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_L^4 \gamma^\mu U_{4\alpha}^L e_L^\alpha + \text{h.c.} . \tag{75}$$

Therefore the ν^4 decay is mostly determined by U_{4a}^L, $a = 1, 2, 3$, i.e. only the mixings in the charged sector are important. Moreover, since $c_l^i \simeq c_l$ for $i = 1, 2, 3$, the model predicts that all the branching ratios in the decay $\nu^4 \to lW$ are approximately equal. We have verified this result in our numerical scan over the parameter space. This has important consequences for collider searches.

An important aspect of these models is the strong coupling between the fourth generation zero modes and the KK excitations of the gauge bosons. Thus, the zero-mode leptons of the fourth generation are expected to be strongly coupled to the KK excitations of the electroweak gauge bosons. The couplings of the zero-mode leptons to the KK excitations of the SM gauge bosons: $W^{(1)\pm}, Z^{(1)}$ and $\gamma^{(1)}$ are the same as the SM gauge couplings of the lighter three generations, up to the enhancement resulting from the zero-mode IR localization, and small flavor-violating corrections. On the other hand, the model has one more neutral gauge boson, the $Z'^{(1)}$, which has no zero mode since it corresponds to the broken-generator combination in the breaking $SU(2)_R \times U(1)_X \to U(1)_Y$. The couplings of zero-mode leptons, again in units of the localization enhancement factor, are given in Table 1, where g_R and g_X are the $SU(2)_R$ and $U(1)_X$ 5D gauge couplings divided by $\sqrt{L}$ so as to render them dimensionless.

In addition, there are two charged $SU(2)_R$ KK gauge bosons, $R_\mu^{\pm(1)}$, resulting in couplings of the zero-mode leptons to KK leptons. Of particular interest are the charged couplings of

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Zero Mode	E1	E2	E3	E4	E5
N_L	$\frac{1}{2}\frac{g_X^2}{\sqrt{g_R^2+g_X^2}}$	$\frac{g_R^2/2+g_X^2}{\sqrt{g_R^2+g_X^2}}$	$\frac{g_R^2/2+g_X^2}{\sqrt{g_R^2+g_X^2}}$	$-\frac{1}{2}\frac{g_R^2}{\sqrt{g_R^2+g_X^2}}$	$-\frac{1}{2}\frac{g_R^2}{\sqrt{g_R^2+g_X^2}}$
N_R	$\frac{1}{2}\sqrt{g_R^2+g_X^2}$	$\sqrt{g_R^2+g_X^2}$	$\sqrt{g_R^2+g_X^2}$	0	0
E_L	$\frac{1}{2}\frac{g_X^2}{\sqrt{g_R^2+g_X^2}}$	$\frac{g_R^2/2+g_X^2}{\sqrt{g_R^2+g_X^2}}$	$\frac{g_R^2/2+g_X^2}{\sqrt{g_R^2+g_X^2}}$	$-\frac{1}{2}\frac{g_R^2}{\sqrt{g_R^2+g_X^2}}$	$-\frac{1}{2}\frac{g_R^2}{\sqrt{g_R^2+g_X^2}}$
E_R	$\frac{1}{2}\frac{g_X^2-g_R^2}{\sqrt{g_X^2+g_R^2}}$	$\frac{g_X^2}{\sqrt{g_X^2+g_R^2}}$	$\frac{g_X^2}{\sqrt{g_X^2+g_R^2}}$	$-\frac{g_R^2}{\sqrt{g_X^2+g_R^2}}$	$-\frac{g_R^2}{\sqrt{g_X^2+g_R^2}}$

Table 1: *Couplings of zero-mode leptons to the Z', in units of the dimensionless enhancement from localization.*

zero-mode fourth-generation leptons to a light KK lepton. For instance, in Model 1 the zero-mode right-handed neutrino $\nu_R(++)$ in (7) couples through the charged right KK mode $R_\mu^\pm$ to the light KK mode $e'_R(-+)$ with strength $g_R/\sqrt{2}$. This is also the case for the coupling between the zero-mode right-handed electron $e_R(++)$, which couples through the charged right current to its $SU(2)_R$ partner ν''_R. Similar charged couplings exist in Models 2, 3, 4 and 5, whenever the zero-mode right-handed neutrino $\nu_R(++)$ or electron $e_R(++)$ belong to a triplet of $SU(2)_R$. These interactions are of interest since, as shown in Section 4.1, the right-handed KK leptons with $(-+)$ boundary conditions are lighter than the typical KK mass scale. Thus, due to the strong coupling of the fourth-generation lepton zero-modes to both the KK gauge bosons and fermions, the single production of a lepton KK mode in association with a fourth-generation zero-mode lepton is a potentially important signal for these models.

Given the couplings above, we can study the phenomenology of fourth-generation leptons at the LHC in the models presented here. The fourth-generation zero-mode leptons will be produced via the s-channel exchange of the SM gauge bosons, as well as the KK gauge bosons including the photon, W and Z KK excitations $A^{(1)}$, $W^{\pm(1)}$ and $Z^{(1)}$, as well as the KK excitation coming from the combination of the generators T_R^3 and X broken by the boundary conditions on the IR brane, $Z'^{(1)}$. They will also be produced through an s-channel Higgs.

Finally, as it was shown in Section 3, there are lepton flavor violating interactions with the Z and the heavier neutral resonances. These interactions allow, in principle, the production of a charged lepton of the fourth generation e^4 together with a light SM charged lepton through an s-channel neutral vector Z or $Z^{(n)}$. Since all the left-handed light leptons have equal localization, the interaction $\bar{e}_L^4 Z e_L^i$, $i = 1, 2, 3$, has the same strength for any flavor (a similar result holds for $Z^{(n)}$). Thus the production process $Z, Z^{(n)} \to \bar{e}_L^4 e_L^i$ and the decay $e_L^4 \to Z e_L^i$ are almost flavor independent. On the other hand, the right-handed interactions $\bar{e}_R^4 Z e_R^i$ are stronger for heavier leptons e^i, thus for neutral right-handed currents the dominant production mechanism is $Z, Z^{(n)} \to \bar{e}_R^4 \tau_R$ and the dominant neutral decay is $e_R^4 \to Z \tau_R$. The flavor-violating couplings, however, are likely to be too small to be observed at the early stages of the LHC [25].

4.3 ν_R^4 phenomenology

Although a detailed phenomenological study will be done elsewhere [26], we make here some general remarks regarding the right-handed zero-mode neutrino, ν_R^4. We start with a brief discussion of the production of ν_R^4 at hadron colliders. In Models 1, 2 and 3 right-handed neutrinos can be pair-produced through their couplings to $Z'^{(1)}$, shown in Table 1. This is not the case in Models 4 and 5. However, since the Yukawa couplings of the fourth-generation neutrinos are large, the ν_R^4 can be produced as a decay product of the Higgs, through $gg \to h \to \nu_L^4 \nu_R^4$. For a heavy Higgs such that $m_h > 2m_{\nu^4}$, with $m_{\nu^4} \sim \mathcal{O}(300)$ GeV, we obtain a branching ratio $BR(h \to \nu_L^4 \nu_R^4) \sim (2-5)\%$, for $m_h \sim (650 - 900)$ GeV. Adding the fact that the presence of a full fourth generation increases significantly the Higgs production cross section by gluon fusion over the usual SM value [6], this production mechanism for ν_R^4 cannot be neglected, not just in Models 4 and 5, but also in Models 1-3 [26].

Finally, in all cases ν_R^4 decays promptly through the Dirac-mass mixing with ν_L^4 into a W and a light charged lepton. Additionally, in embeddings 1-4 (see Section 2.1), where the ν_R is charged under the $SU(2)_R$, the ν_R^4 decay can proceed through a virtual charged KK vector $R^{\pm(n)}$ and a virtual KK charged fermion $e'^{(n)}$ of the fourth generation. $R^{\pm(n)}$ mixes via Higgs mass insertions with the zero mode $W^{(0)}$ and $e'^{(n)}$ mixes with the charged zero-mode fermions $e^{i(0)}$. However, this modes will be suppressed by KK masses compared to the one mentioned above.

5 Conclusions

Extensions of the standard model with a fourth generation must address the question of why the fourth generation neutrinos are so much heavier than the three-generation light neutrinos. In addition, they must satisfy constraints from lepton flavor violation and reproduce the pattern of mixings and mass differences of light neutrinos. We have addressed these questions in a class of models in the context of theories with warped extra dimensions. In this scenario both the charged lepton and the neutrino are heavy as a consequence of their localization in the extra dimension. They are both IR-localized which results in a large overlap with the Higgs, independently of whether this is an elementary scalar or an effective degree of freedom resulting from the condensation of fourth-generation quarks. The models naturally have a UV-localized Majorana mass term which results in a see-saw mechanism only efficient with neutrinos with zero-mode wave-functions localized close to the UV brane. Thus, a consequence of this setup is that the light neutrinos of the first three generations are mostly Majorana particles since they have a significant overlap with the UV brane and therefore feel the effect of the Majorana mass term. On the other hand, fourth-generation neutrinos are IR-localized and will be mostly Dirac particles, since they have little overlap with the Majorana mass term.

The exception to this is the case where right-handed neutrinos come from singlets under the bulk gauge symmetries (embedding 5 in Section 2.1). As a consequence, in addition to the UV-localized Majorana term present in embeddings 1-4, there is an IR-localized Majorana mass that only affects significantly the fourth generation neutrinos. The two Majorana components are split with an order-one splitting, resulting in a potentially light Majorana fourth-generation neutrino that could be as light as 100 GeV.

For all the possible embeddings, we have shown that in order to satisfy the existing constraints from neutrino mixings and in combination with the lepton spectrum the localization of the lepton left-handed zero-mode doublets must be almost degenerate for the first three generations, i.e. $c_\ell^i = c_\ell$, for $i = 1, 2, 3$. Deviations from the equality cannot significantly exceed 1% in order to avoid a hierarchical mixing pattern in the light neutrino sector. Similarly, the localization of the right-handed zero-mode neutrinos of the light three generations is chosen to be the same in order to match the see-saw picture with a single suppression scale. This suggests the presence of a flavor symmetry acting on UV-localized left-handed leptons and right-handed neutrinos. We do not need to impose that the bulk masses of the right-handed charged leptons be the same for the light generations. This freedom allows us to obtain the correct spectrum.

The presence of flavor-violating interactions leads to several lepton flavor-violating processes. We studied the impact of the experimental bounds on lepton flavor violating processes on the parameter space of the model. We have found that, for generic embeddings, the KK mass scale is bound to be $m_{KK} \gtrsim 4$ TeV by the experimental limits on $\mu^- \to e^- e^+ e^-$, whereas considering the rate of $\mu^- \to e^-$ conversion results in $m_{KK} \gtrsim 6$ TeV, although there are some small regions of parameter space where the FCNC effects are minimized and $m_{KK} \sim 2.4$ TeV is allowed. We have shown that in two of the five embeddings proposed in Section 2.1, those we called Model 4 and 5, it is possible to evade the lepton flavor violation bounds even with KK masses as low as $m_{KK} \simeq 2.4$ TeV, for larger regions of the parameter space. These embeddings realize a custodial symmetry that protects the Z couplings to leptons.

The parameter space of the models is almost entirely determined by the bulk mass parameters of the zero-mode leptons. This fixes the zero-mode mass spectrum of the fourth generation as well as its couplings to the KK gauge bosons and the Higgs, through the enhancement factor resulting from IR localization. This, together with the gauge couplings from Table 1, can be used to study the phenomenology of the fourth-generation lepton sector at colliders. As shown in the Table, these gauge couplings depend on the embedding of the leptons in the 5D gauge theory. We considered five different possible embedding in Section 2.1. Although the gauge couplings of the zero-mode leptons to the SM electroweak gauge bosons and their KK excitations do not depend on the 5D embedding, their couplings to the $Z'^{(1)}$, the KK mode of the broken generator in $SU(2)_R \times U(1)_X \to U(1)_Y$, do as it can be seen in Table 1. Furthermore, in Model 5, the right-handed neutrino has no 5D gauge couplings. Thus, in this embedding, the fourth-generation ν_R^4 couples only through the Higgs sector. This implies that both its production and decay must involve its Yukawa couplings. The dominant production of ν_R^4 is then through the s-channel Higgs, with the decay proceeding through the mass mixing with ν_L^4.

Another important prediction is the appearance of light KK leptons, as described in Section 4.1. As shown in Figures 3, 4 and 5, KK leptons with $(-+)$ boundary conditions can be as light as 0.5 TeV and are generically considerably lighter than the KK gauge bosons. One important consequence is that it is more kinematically favorable to singly produce a KK lepton in association with a zero-mode lepton, particularly if this is a fourth generation lepton since the corresponding effective gauge coupling would be enhanced due to its IR localization. This and other aspects of the phenomenology of the lepton sector of a fourth-generation at the LHC, including its production and decay, will be studied in detail in a future publication [26].

A   Perturbative diagonalization of the neutrino mass

ν contains four generations of neutrinos, we split them in the following way:

$$\nu^t = (\nu^e, \nu^\mu, \nu^\tau, \nu^4) = (\nu^a, \nu^4) , \qquad a = 1, 2, 3 , \tag{76}$$

thus Latin indexes label the light generations. Using this notation we can write the Dirac and Majorana mass matrices as:

$$M_{RR}^\nu = \begin{bmatrix} M_{RR}^{ab} & M_{RR}^{a4} \\ M_{RR}^{4b} & M_{RR}^{44} \end{bmatrix} , \qquad M_{LR}^\nu = \begin{bmatrix} M_{LR}^{ab} & M_{LR}^{a4} \\ M_{LR}^{4b} & M_{LR}^{44} \end{bmatrix} . \tag{77}$$

We integrate out the heavy neutrinos ν_R^a at tree level, and obtain the effective mass term of Eq. (32), with $M_{\rm eff}$ given by:

$$M_{\rm eff} = \begin{bmatrix} -M_{LR}^{ab}(M_{RR}^{bc})^{-1}M_{RL}^{cd} & -M_{LR}^{ab}(M_{RR}^{bc})^{-1}M_{RL}^{c4} & M_{LR}^{a4} - M_{LR}^{ab}(M_{RR}^{bc})^{-1}M_{RR}^{c4} \\ -M_{LR}^{4b}(M_{RR}^{bc})^{-1}M_{RL}^{cd} & -M_{LR}^{4b}(M_{RR}^{bc})^{-1}M_{RL}^{c4} & M_{LR}^{44} - M_{LR}^{4b}(M_{RR}^{bc})^{-1}M_{RR}^{c4} \\ M_{RL}^{4d} - M_{RR}^{4b}(M_{RR}^{bc})^{-1}M_{RL}^{cd} & M_{RL}^{44} - M_{RR}^{4b}(M_{RR}^{bc})^{-1}M_{RL}^{c4} & M_{RR}^{44} - M_{RR}^{4b}(M_{RR}^{bc})^{-1}M_{RR}^{c4} \end{bmatrix} . \tag{78}$$

We can see that most of the entries of $M_{\rm eff}$ are suppressed by the see-saw mechanism. Moreover, since ν_R^4 is localized towards the IR and the right-handed Majorana mass is localized in the UV, M_{RR}^{44} is very suppressed also (one can check that for the range of parameters interesting for the phenomenology, M_{RR}^{44} is several orders of magnitude smaller than the entries not suppressed by the see-saw). Therefore, at leading order, we obtain an effective theory with two heavy Majorana states described by Eqs. (35) and (36). At this level of approximation, there is a degenerate subspace o dimension 3 with eigenvalues equal to zero, corresponding to three massless neutrinos. These massless eigenstates have no projection on the fifth component at this order. Introducing a spurious infinitesimal ϵ we can compute the light eigenvalues at leading order:

$$\det(M_{\rm eff}^\epsilon - \epsilon\lambda) = 0 , \qquad M_{\rm eff}^\epsilon = \begin{bmatrix} \epsilon S^{ij} & M^{i4} \\ M^{4i} & \epsilon T \end{bmatrix} , \qquad i, j = 1, \ldots 4 , \tag{79}$$

where we explicitly show the suppressed entries of $M_{\rm eff}$. At leading order in ϵ (*i.e.*: $\mathcal{O}(\epsilon^3)$) this equation has three solutions. Once we obtain the light eigenvalues λ^a, we can write, at leading order, a explicit expression for the light eigenvectors in terms of the 4×4 matrix S and the dimension 4 vector M^{i4}. For simplicity we split the light eigenvectors isolating the fifth component:

$$v^a = \begin{bmatrix} v_i^a \\ v_5^a \end{bmatrix} , \qquad a = 1, 2, 3 , \ i = 1, \ldots 4 , \tag{80}$$

where a labels the light eigenvectors and i labels the first four components of each vector. Thus v_i^a and v_5^a are given by the following equations in a space of dimension 4:

$$|v_5^a|^2 = \left(M^\dagger[(S - \lambda^a)^{-1}]^\dagger(S - \lambda^a)^{-1}M\right)^{-1} , \tag{81}$$

$$v_i^a = -v_5^a[(S - \lambda^a)^{-1}M]_i , \tag{82}$$

where, although we have not explicitly shown the indexes, $M = M^{i4}$ is a vector of dimension 4, and $(S - \lambda^a)^{-1}$ is the inverse of matrix $(S - \lambda^a)$, acting on M.

B   Scan

There is a large set of solutions giving the right spectrum and mixings in the leptonic sector, together with a heavy fourth generation. As discussed in Section 2.3, there are also some additional constraints arising from the phenomenology, like $\mu \to e\gamma$. We show here a set of 5D parameters satisfying all these constraints, with light neutrino mass splittings given by:

$$\Delta m_{\text{sol}}^2 \simeq 1.1 \ 10^{-4}\text{eV}^2 , \qquad \Delta m_{\text{atm}}^2 \simeq 2.6 \ 10^{-3}\text{eV}^2 . \tag{83}$$

We have scanned over the parameter space, considering real symmetric Yukawas and the identity for λ_R in order to simplify the computation. The 5D Yukawa matrices are approximately given by:

$$\lambda_e = \begin{bmatrix} 2.5 & 1.1 & 0.6 & -0.6 \\ 1.1 & 1.1 & -0.5 & -0.6 \\ 0.6 & -0.5 & -0.1 & -1.4 \\ -0.6 & -0.6 & 1.4 & 1.1 \end{bmatrix} , \quad \lambda_\nu = \begin{bmatrix} -2.4 & 0.8 & -1.1 & -0.6 \\ 0.8 & 3.9 & -1.5 & -1.4 \\ -1.1 & -1.5 & 2.8 & 0.7 \\ -0.6 & -1.4 & 0.7 & 1.1 \end{bmatrix} , \quad \lambda_R = 1_{4\times4} . \tag{84}$$

The spectrum of zero modes reproduces the charged leptons of the SM, with heavy leptons: $m_\nu^4 \simeq m_e^4 = 300$ GeV, and light neutrinos with masses approximately given by [12]:

$$m_\nu^1 = 18 \text{ meV} , \qquad m_\nu^2 = 53 \text{ meV} , \qquad m_\nu^3 = 54 \text{ meV} . \tag{85}$$

The corresponding mixing matrix V is given by:

$$V = \begin{bmatrix} 0.82 & -0.34 & 0.46 & -0.04 \\ 0.57 & 0.39 & -0.72 & 0.03 \\ 0.06 & 0.85 & 0.52 & 0.01 \\ -0.01 & 0.02 & -0.02 & -0.71 \\ -0.01 & 0.02 & -0.02 & -0.71 \end{bmatrix} , \tag{86}$$

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